Unit 6H: Solve Systems with Matrices Study Guide Name: ____________________ Per: ____

SHOW YOUR WORK FOR FULL CREDIT. NO WORK, NO CREDIT. NO WORK IN PEN.

Assn	Learning Objective	A Day	B Day	Done
6SG	Matrix System Study Guide			
6.1	Create a Matrix from a System & vice versa	Nov 6	Nov-7	
6.2	Row Echelon Reduction	Nov-8	Nov 9	
6.3	Matrices of Steroids	Nov 12	Nov 13	
6.4	More Matrices on Steroids	Nov 14	Nov 15	
	Unit 6 EMT	Nov 16	Nov 19	

Targets	Sample	Ugh	Almost	Got it!	Assn
Create an augment matrix from the system.	$\begin{cases} y + \frac{3}{5}x = 3 \\ y = -\frac{10}{5}x + 6 \end{cases}$				6.1
Solving Systems of Equations with Matrices	Given $\begin{cases} y + \frac{3}{5}x = 3 \\ y = -\frac{10}{5}x + 6 \end{cases}$, rewrite as an augmented matrix.				6.2, 6.3
Solve system using Row Echelon Reduction	Solve the following matrix using row echelon reduction. $\begin{bmatrix} 3 & 2 & -6 \\ 1 & 2 & 2 \end{bmatrix}$				6.2, 6.3
Solve a system of equation from a story problem using row echelon.	Rachel and Zack purchase school supplies. Rachel buys 3 notebooks and 5 packages of pencils for $22.75. Zack buys two notebooks and 3 packs of pencils for $14.25. What's the cost of both items?				6.2, 6.3

Vocabulary

Augmented Matrix____

Elementary Row Operations:____________________________________

Row Echelon ____

Additive Inverse____

Multiplicative Inverse____

Multiplicative Identity ____

Create a Matrix from a Story Problem.

Systems of equations can be solved using _________________ or elimination. **Augmented matrices** can be solved like a ____________ of equations. Write the system as described in Unit 3 Systems. If an equation is in slope-intercept _________, rewrite it in standard form aligning the coefficients as if for elimination in Standard Form (A*x* + B*y* = C). (It does not matter which variable comes first but must be the same order for both equations.). In elimination, make one of the coefficients "0" to solve for the other variable. Write the coefficients of the ________________ into an augmented matrix.

Rachel buys 2 notebooks and 3 packages of pencils for $12. Zack buys four notebooks and returns 2 packs of pencils and pays $20. How much does a notebook and package of pencils cost?

Complete the equations for the story problem above. $\begin{cases} \_\_\_x + 3y = \_\_\_\_ \\ 4x - \_\_\_y = \_\_\_\_ \end{cases}$

Below shows one way to solve comparing Elimination to Row Echelon Reduction of the above.

System of Equations	Process	Matrix
$\begin{cases} 2x+3y=12 \\ 4x-2y=20 \end{cases}$	1. Place the constants and answers from the equations in Standard Form into the rows of an ________________ matrix.	$\begin{bmatrix} 2 & 3 & 12 \\ 4 & -2 & 20 \end{bmatrix}$
$\begin{cases} -4x-6y=-24 \\ 4x-2y=20 \end{cases}$	2. Eliminate the number in the next row by multiplying the first row (r_1) by a number that would add/subtract with the next row.	$\begin{bmatrix} -4 & -6 & -24 \\ 4 & -2 & 20 \end{bmatrix}$
$\begin{cases} 2x+3y=12 \\ 0x-8y=-4 \end{cases}$	3. Add/Subtract the two ________ together to eliminate the first element in the second row.	$\begin{bmatrix} 3 & 4 & 12 \\ 0 & -8 & -4 \end{bmatrix}$
$\begin{cases} 1x+1.5y=6 \\ 0x-8y=-4 \end{cases}$	4. Divide the first row by ___ to make the coefficient of x equal to ___.	$\begin{bmatrix} 1 & 1.5 & 6 \\ 0 & -8 & -4 \end{bmatrix}$
$\begin{cases} 1x+1.5y=6 \\ 0x+1y=\frac{1}{2} \end{cases}$	5. Make the second element in the second row/second column into a ____ by multiplying or dividing.	$\begin{bmatrix} 1 & 1.5 & 6 \\ 0 & 1 & \frac{1}{2} \end{bmatrix}$
$\begin{cases} 1x+1.5y=6 \\ 0x-1.5y=-.75 \end{cases}$	6. Multiply or divide the second row to make the elements the same in the first row __________ column.	$\begin{bmatrix} 1 & 1.5 & 6 \\ 0 & -1.5 & -.75 \end{bmatrix}$
$\begin{cases} 1x+0y=4.25 \\ 0x-1.5y=-.75 \end{cases}$	7. Add/subtract the rows together to eliminate the element in the ________ row second column.	$\begin{bmatrix} 1 & 0 & 4.25 \\ 0 & -1.5 & -.75 \end{bmatrix}$
$\begin{cases} 1x+0y=4.25 \\ 0x+1y=.50 \end{cases}$	8. Divide the second row to make the second element equal ___. The first row of this matrix shows that one x and 0 y's equals 4.25. The second row shows that 0 x's and one y and equals 0.5.	$\begin{bmatrix} 1 & 0 & 4.25 \\ 0 & 1 & 0.50 \end{bmatrix}$
x = 4.25, y = .50	9. Notebooks cost ________ each and ____________ cost $0.50.	x = 4.25, y = .50

Matrices can be used to solve any size system. For example, a system with 3 equations with 3 ___________ would make (with answers) a 3 X ___ augmented matrix. Regardless of the size, it is often easiest to line up the "1's" in a diagonal with the zeros where the first column represents the x's, the second the y's, the third, the z's, etcetera.

Dependent Systems (Infinite Solutions)

- When the equations are added/subtracted, all elements will equal ____.
- This system of equations will have __________ solutions.

Given the system $\begin{cases} 12x+3y=6 \\ 6x+1.5y=3 \end{cases}$, write it in a matrix and solve by row echelon reduction:

$$\begin{bmatrix} 12 & 3 & 6 \\ 6 & 1.5 & 3 \end{bmatrix} \Rightarrow \begin{bmatrix} 12 & 3 & \_ \\ 12 & \_ & 6 \end{bmatrix} \Rightarrow \begin{bmatrix} 0 & 0 & 0 \\ \_ & 0 & 0 \end{bmatrix}$$

Inconsistent Systems (Parallel Lines)

- When solving, all coefficients will become zeros because they have the _________ slope.
- The "answer" column will have different values which indicates there are different y-______________
- The system of equations will have _________ solutions.

Given: $\begin{cases} 12x+3y=8 \\ 6x+1.5y=3 \end{cases}$, write the augmented matrix and solve using row echelon reduction.

$$\begin{bmatrix} 12 & \_\_ & 8 \\ \_\_ & 1.5 & \_\_ \end{bmatrix} \Rightarrow \begin{bmatrix} 12 & 3 & \_\_ \\ \_\_ & 3 & 6 \end{bmatrix} \Rightarrow \begin{bmatrix} 0 & 0 & 2 \\ 0 & 0 & 0 \end{bmatrix}$$

Solving a system of equations using matrices practice.

Two students solved the following system using matrices. $\begin{cases} 3x+y=7 \\ 4x+2y=10 \end{cases}$

Abby solved it this way. Explain or write each step she took.

$\begin{bmatrix} 3 & 1 & 7 \\ 4 & 2 & 10 \end{bmatrix}$

$\begin{bmatrix} \quad & \quad & \quad \\ \quad & \quad & \quad \end{bmatrix}$

$\begin{bmatrix} 3 & 1 & 7 \\ -2 & 0 & -4 \end{bmatrix}$

$\begin{bmatrix} 3 & 1 & 7 \\ 1 & 0 & 2 \end{bmatrix}$

$\begin{bmatrix} \quad & \quad & \quad \\ \quad & \quad & \quad \end{bmatrix}$

$\begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 2 \end{bmatrix}$

Show a different way to solve for the same answer.

$\begin{bmatrix} 3 & 1 & 7 \\ 4 & 2 & 10 \end{bmatrix}$

Explain how your matrix was different:____

Solving Systems with Inverse Matrices

Multiplicative inverses make 1's. The inverse of 2 is $\frac{1}{2}$. The inverse of $-\frac{3}{4}$ is $-\frac{4}{3}$. Given $2x = 10$, to make the coefficient 2 into the multiplicative identity 1, multiply both sides of the equation by the multiplicative inverse $(\frac{1}{2})$.

The system $\begin{cases} 3x + 2y = 6 \\ x + 4y = 2 \end{cases}$ can be written as the matrix equation $\begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 6 \\ 2 \end{bmatrix}$. This system could be solved by multiplying by the ____________ of $\begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix}$. Multiplying the coefficient matrix $\begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix}$ by its inverse would result in the identity matrix $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$. This would mean that the coefficient of x in the first equation is now 1 and the coefficient of the y is 0. The coefficient of x in the second equation is 0 and the coefficient of the y is 1. Aligning these with the answer matrix (after multiplying by the inverse) gives the solution to the system.

Finding Inverse Matrices

To solve the system, find the multiplicative identity $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$. Using the system above, multiply $\begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix}$ by $\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$. This gives two new systems $\begin{cases} 3a + \_\_\_ = 1 \\ \_\_\_ - c = 0 \end{cases}$ and $\begin{cases} \_\_\_ + 2d = 0 \\ b - \_\_ = \_\_ \end{cases}$. Solve these two systems for *a, b, c,* and *d* using matrices.

Plugging your solutions for *a, b, c,* and *d* into the matrix yields $\begin{bmatrix} \frac{2}{5} & \frac{-1`}{5} \\ \frac{-1}{10} & \frac{3}{10} \end{bmatrix}$. Multiply both sides of the matrix equation by the inverse matrix. **SYW.**

$$\begin{bmatrix} \frac{2}{5} & \frac{-1`}{5} \\ \frac{-1}{10} & \frac{3}{10} \end{bmatrix} \begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \frac{2}{5} & \frac{-1`}{5} \\ \frac{-1}{10} & \frac{3}{10} \end{bmatrix} \begin{bmatrix} 6 \\ 2 \end{bmatrix}.$$

Write your final solution. ___________

Examine the inverse matrix $\begin{bmatrix} \frac{2}{5} & \frac{-1`}{5} \\ \frac{-1}{10} & \frac{3}{10} \end{bmatrix}$. Note that each of the elements of the matrix have a divisor of 10. Factoring out $\frac{1}{10}$ from each element leaves $\frac{1}{10}\begin{bmatrix} 4 & -2 \\ -1 & 3 \end{bmatrix}$. (Multiplied together, it yields the inverse.) The 10 in the $\frac{1}{10}$ is called the determinant. How could you quickly calculate the determinant? (See your notes from class.)

The matrix $\begin{bmatrix} 4 & -2 \\ -1 & 3 \end{bmatrix}$ is called the **Adjunct, Adjoint, or Adjugate Matrix.** How do the numbers in the Adjunct Matrix relate to your original matrix from your system?

Using the hints/short cut above, find the inverse matrix for

$\begin{bmatrix} 2 & 9 \\ 5 & 4 \end{bmatrix}$ $\begin{bmatrix} 3 & 2 \\ 7 & 5 \end{bmatrix}$ $\begin{bmatrix} 6 & 4 \\ 3 & 2 \end{bmatrix}$

Inverse Matrices Practice

Solve the following systems using Inverse Matrices.

$\begin{cases} 5x - 6y = -14 \\ 3x + 4y = 22 \end{cases}$ $\begin{cases} 2x + 4y = 6 \\ 4x - 5y = -22 \end{cases}$